Partial Torus Instability

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Abstract

Flux ropes are now generally accepted to be the magnetic configuration of Coronal Mass Ejections (CMEs), which may be formed prior or during solar eruptions. In this study, we model the flux rope as a current-carrying partial torus loop with its two footpoints anchored in the photosphere, and investigate its stability in the context of the torus instability (TI). Previous studies on TI have focused on the configuration of a circular torus and revealed the existence of a critical decay index of the overlying constraining magnetic field. Our study reveals that the critical index is a function of the fractional number of the partial torus, defined by the ratio between the arc length of the partial torus above the photosphere and the circumference of a circular torus of equal radius. We refer to this finding the partial torus instability (PTI). It is found that a partial torus with a smaller fractional number has a smaller critical index, thus requiring a more gradually decreasing magnetic field to stabilize the flux rope. On the other hand, the partial torus with a larger fractional number has a larger critical index. In the limit of a circular torus when the fractional number approaches one, the critical index goes to a maximum value that depends on the distribution of the external magnetic field. We demonstrate that the partial torus instability helps us to understand the confinement, growth, and eventual eruption of a flux rope CME.

Objective

- Revisit the analytical justification of the torus instability and the derivation of the critical index n when applying a geometrical assumption to describe the evolution of the major radius $R(Z)$

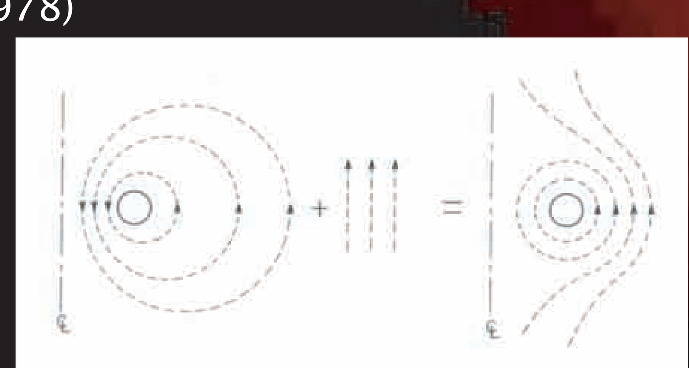
- Show through a model run how this new result helps to better understand the confinement, growth, and eventual eruption of a flux rope CME.

- Demonstrate, with an example eruptive prominence event, how an observational test of the theory may proceed

Torus Instability

- This type of instability arises when considering the stability of a plasma ring with toroidal current. When Lorenz forces dominate, stability of such a plasma ring can be established with an external magnetic field that is perpendicular to the axis of the torus (Shafranov 1966)

- The figure below shows a cross section of the torus; the LHS shows the magnetic field due to the toroidal current plus a perpendicular external field, the RHS indicates that the system is in balance (Figure taken from Bateman 1978)



- However, if the external field decreases rapidly enough in the direction of the major radius then any outward perturbation will cause the inward Lorentz force to decrease faster than the outward Lorentz force resulting in the expansion of the ring (Bateman 1978)

- The external magnetic field B_s is characterized by a decay index n, and if $n > 3/2$ the torus becomes unstable and outward magnetic forces dominate causing expansion

$$n \equiv -\frac{Z}{B_s}\frac{dB_s}{dZ}$$

- Observations of potential coronal field (Liu 2008) and simulations of erupting flux ropes (Kliem & Török 2006, Török & Kliem 2007, Fan & Gibson 2007) indicate that the value of n may be between 1.5 and 2

Flux Rope Model Analysis

This new analysis proceeds like those of previous authors (e.g. Bateman 1978, Chen 1989, Cargill 1994, Kliem & Török 2006). A loop is assumed consisting of two current channels perpendicular to each other (i.e. flux rope). One in the toroidal direction, and the other in the poloidal direction. The loop is described by a major radius R and minor radius a, and it is assumed that $R >> a$. The flux rope is modeled as a partial torus characterized by a parameter Θ defined such that $2\pi\Theta R$ is the arc length of the torus above the photosphere. The overlying field B_s is perpendicular to the loop axis and parallel to the surface. It is also assumed that plasma beta is less than one and that Lorentz forces dominate. The equations of motion can be written;

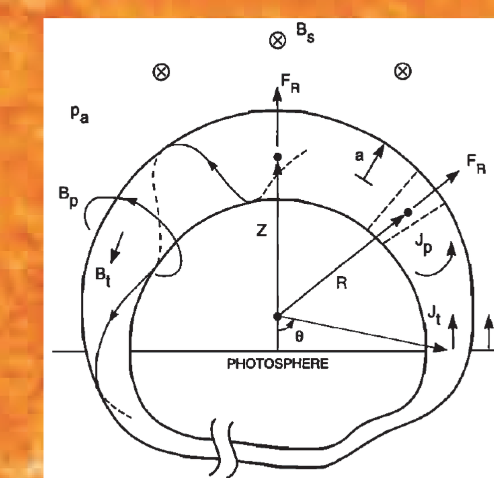


Figure from Chen (1996) showing flux rope topology

for the apex height Z

$$M\frac{d^2Z}{dt^2} = \frac{I_t^2}{c^2R}\left[\left(ln\left(\frac{8R}{a}\right) - 2 + \frac{\xi_i}{2}\right) - \frac{1}{2}\frac{B_t^2}{B_p^2} + 2\frac{R}{a}\frac{B_s}{B_p} + 1\right]$$

and the minor radius

$$M\frac{d^2a}{dt^2} = \frac{I_t^2}{c^2a}\left[\frac{B_t^2}{B_p^2} - 1\right]$$

An equation to describes the evolution of R, as a function of the apex Z, was first proposed by Chen (1989) and is key to this analysis

$$R = \frac{Z^2 + S_0^2}{2Z}.$$

where S_0 is the half footpoint separation, which is assumed fixed.

The analysis proceeds by linearizing the equations of motion about a perturbation δZ at equilibrium. The expression reduces to

$$\frac{d^2\delta Z}{dt^2} = \Gamma^2\delta Z$$

where Γ^2 is a function of n and the sign of Γ^2 determines the behavior of the system during a perturbation.
- If $\Gamma^2 < 0$ then oscillatory and stable
- If $\Gamma^2 > 0$ then exponentially divergent indicating instability

Partial Torus Instability

Define Z_{ct} and n_{ct} as the critical values when $\Gamma^2(n(Z_{ct}) = n_{ct}) = 0$, which is the boundary between stability and instability. Rearranging Γ^2 for n_{ct} the following expression is found

$$n(Z_{ct}) = n_{ct} = \frac{2Z_{ct}}{R}\frac{dR}{dZ} + \frac{Z_{ct}}{\Theta}\frac{d\Theta}{dZ} + \frac{Z_{ct}}{cL_pI_t}\frac{d\Phi_s}{dZ},$$

$$\Phi_s = \int B_s \cdot dA$$

where Φ_s is the flux between the loop apex and the surface, and L_p the inductance of the Loop. *Note that the equation of n_{ct} is charachterized by the scale length S_0

This result is a prediction of this flux rope model that says that the critical decay index is dependent on the geometrical shape of the flux rope $n_{ct} = n(Z_{ct})$

Plot of n_{ct} showing 3 possible solutions
- Solid line assumes that

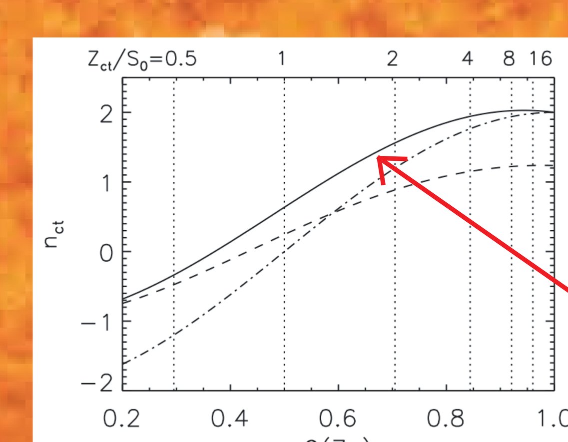
$$d\Phi_s/dZ = 0$$

- Dash line assumes that

$$d\Phi_s/dZ \neq 0$$

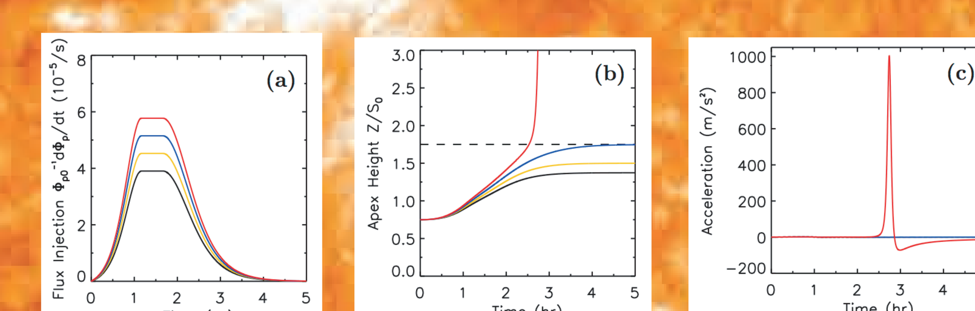
- Dot-dashe line assumes self-similar evolution $R/a = const$ and n_{ct} reduces to

$$n_{ct} = \frac{2Z_{ct}}{R}\frac{dR}{dZ}$$



The significance of this result is that if the index of the external field is less than the critical value the flux rope will be stable to perturbation. On the other if the index is greater than the critical value the flux rope will be unstable to perturbation.

Model Run

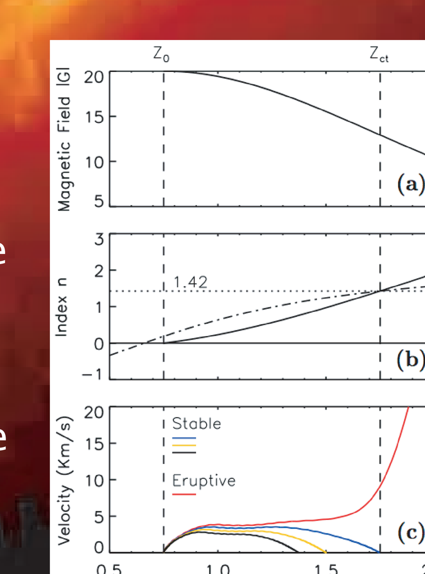


- The full equations of motions that include Lorentz, gravity, and thermal pressure gradient force as outlined in Chen (1996) are solved to demonstrate the theory
- It is assumed that the flux rope is driven by flux injection of unspecified origin

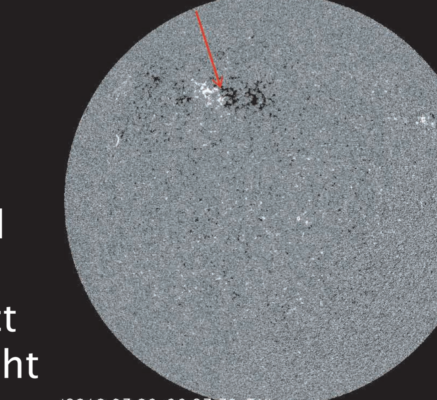
*** Once the flux rope exceeds the critical height Z_{ct} it will subsequently erupt, else it will be confined**

- In this model run $Z_{ct} = 1.75\ S_0$ and $n_{ct}(Z_{ct}) = 1.42$ assuming that $d\Phi_s/dZ = 0$
- The dot-dash line in the middle panel corresponds to the solid line in the plot to the left

- Between Z_0 and Z_{ct} the flux rope is stable since $n < n_{ct}$ and driven by flux injection, once $Z > Z_{ct}$ and $n > n_{ct}$ the the flux rope erupts



Observational Test



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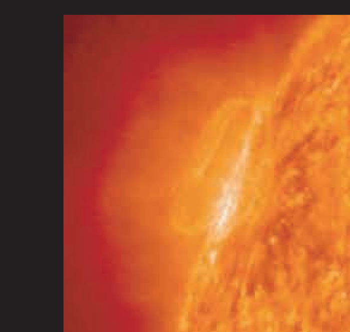
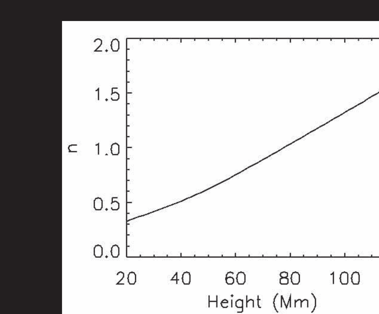
An observational test may be done by examing the magnetic field profile of an event during its eruption and comparing it to the onset height of the event. Though careful interpretation of the data is necessary since direct observations a flux rope have not been made. It is thought that a prominence is supported by a flux rope, hence its evolution could be used as a proxy for a flux rope.

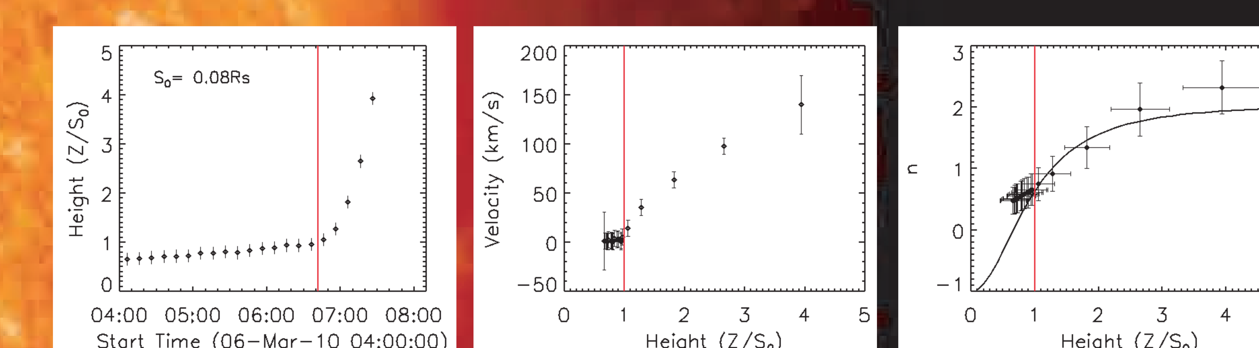


Prominence seen by EUVI A at 2010/03/06 04:06 UTC

The prominence eruption of 2010 March 06 onset time of 06:41 UTC is studied
- The event is seen on the NE from 1AU at the Earth
- Red arrow in the SOHO/MDI image indicates the position of the magnetic inversion line
- STEREO/SECCHI EUVI 304 AHEAD observed this as a prominence on the NE limb
- In the BEHIND spacecraft it appears as a filament very close to the NW limb

- The field perpendicular to the solar surface is the component of interest
- The plot to the right shows the decay index of the extrapolated (PFSS) magnetic field averaged over the neutral line for this event





The figures above show the kinematical evolution triangulated with STEREO A&B
- Solid black line in right plot is the theoretical curve of n_{ct} with $d\Phi_s/dZ = 0$
- Red vertical lines indicate onset
- Onset height of $Z/S_0 = 1$ indicates a semi-circular shape, justifying the simulations of Török & Kliem 2007, Fan & Gibson 2007
- This is a very preliminary result, at this time it is not clear how to interpret these results

Conclusion

The magnetic fields that overly flux ropes play an important role in the eruption or confinement of CMEs.

- The new analysis shows that the critical index is a function of the geometrical shape of the evolving flux rope $n_{ct}(Z_{ct})$

- The importance of this is that once a CME flux rope reaches the height of the critical index n_{ct}, it will become unstable and erupt, if it does not it will be confined. This was demonstrated with a model run, though it is unclear what mechanism drives the flux rope to the critical height

- A preliminary observation to support this theory was shown, in order to have a meaningful result a statistically significant number of event will have to be analyzed. A plot of monotonically increasing values of n vs. Z/S_0 is expected

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Background image - STEREO EUVI A 2010/03/06 07:16:15 UTC